PROBLEMS IN DEPTH PERCEPTION:
A Method of Simulating Objects Moving in Depth

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Visual size is an important variable in a variety of experiments; for example, in studies involving size constancy, in studies of size as a cue to distance, and in studies concerned with the perceptual effects of the optical-expansion pattern associated with objects moving in depth. In many of these experiments, it would be useful to be able to vary continuously the visual angle of complex objects without changing other visual qualities of the objects, such as the stimulus to accommodation. A method of producing controlled and continuous variations in visual size can be achieved by applying principles involved in the “shadow transformer” described by Gibson,\(^1\) or in the projection system described by the de Florez Company.\(^2,3\) The resulting apparatus, which might be called the moving-object simulator, is useful in simulating the motion of complex objects or surfaces in depth.

A schematic side-view diagram that can be used in discussing the principles involved in the moving-object simulator is shown in Figure 1. In Figure 1, \(e_R, f_R, g_R, h_R\) represents a simulated, rigid, physical configuration (object) observed from the position \(O\). By the term “rigid configu-

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![Diagram](image)

**Figure 1.** A side-view schematic diagram for considering factors involved with using a point source of light in the simulation of objects moving in depth.
ration" is meant a configuration whose angular and linear dimensions are constant. The front and rear object surfaces of size \( S_{N_1} \) and \( S_{N_2} \), respectively, are parallel to the frontal plane of the observer (for simplicity of discussion), are separated by a distance \( L_N \) measured along the line OQ perpendicular to a screen W, and are at distances \( D_{N_1} \) and \( D_{N_2} \) from O. The top surface \((f_N, g_N)\) of the object forms an angle \( \beta_N \) with the line OQ. The screen is a distance \( D_0 \) from O along OQ. Straight lines from the observer's position O through \( f_N \) and \( g_N \) intersect the screen at a and b, respectively. At a distance \( D_r \) on the other side of the screen is a point source (P) that transilluminates another configuration \( e_r, f_r, g_r, h_r \) (called the transparency). The front and rear surfaces of the transparency are of sizes \( S_{T_1} \) and \( S_{T_2} \), and are parallel to the screen, are separated from each other by a distance \( L_T \) measured along the line PQ perpendicular to the screen W, and are at distances \( D_{T_1} \) and \( D_{T_2} \) from P, respectively. The top surface \((f_T, g_T)\) of the transparency configuration forms an angle \( \beta_T \) with the line PQ. The screen (W) is a rear-projection screen with, for example, the image of \( f_T \) and \( g_T \) appearing on the screen at a and b, respectively, and being visible from the position of O.

The problem of this paper is to determine the values of \( \beta_T, S_{T_1}, S_{T_2}, \) and \( L_T \) so that, when the transparency \( e_T, f_T, g_T, h_T \) is moved as a rigid object along PQ, it will produce a changing stimulus on the screen and, hence, an optical-expansion pattern in the eye of the observer equivalent to the optical-expansion pattern that would have occurred from the rigid physical configuration \( e_N, f_N, g_N, h_N \) moving toward or away from O (along OQ or an extension of OQ). The simulated configuration \( e_N, f_N, g_N, h_N \) is assumed to be rigid because the purpose of the apparatus is to simulate physically rigid objects. The configuration \( e_T, f_T, g_T, h_T \) must be rigid if the apparatus is to be physically simple. The case in which the configurations have a depth component \( (L_0 \) and \( L_T) \) is selected for discussion because it represents a general formulation of the problem.

It will be noted that, in Figure 1, parts of each of the configurations can obscure other parts of the same configuration. For example, if the surface \( e_N, f_N \) in the simulated configuration were opaque with the surface \( g_N, h_N \) directly behind \( e_N, f_N \), the front surface \( e_N, f_N \) would obscure the remaining surfaces of this configuration. In this case, only the surface \( e_T, f_T \) in the transparency would be required for the simulation. Thus, it is not intended that all parts of Figure 1 necessarily represent an actual simulation. Figure 1 is useful, however, in considering the relations involved in any simulation of a rigid configuration by a rigid transparency using a point-source system of projection. For example, conclusions involving the relation between \( \beta_T \) and \( \beta_N \) or the relation between \( L_T \) and \( L_N \) can be expected to apply respectively to any angles and any depth extents in the configurations. Also, although only the vertical components of frontal extents are represented in Figure 1, conclusions concerning the relation between simulated and transparency configurations with respect to such vertical components can be applied to horizontal components as well.

Using Figure 1, it can be shown that

\[
D_{N_1} = \left( \frac{D_0}{D_P} \right) \left( \frac{S_{N_1}}{S_{T_1}} \right) D_{T_1}
\]

(1)

and

\[
D_{N_2} = \left( \frac{D_0}{D_P} \right) \left( \frac{S_{N_2}}{S_{T_2}} \right) D_{T_2}.
\]

(2)

Or, more generally, if

\[
\frac{S_{N_1}}{S_{T_1}} = \frac{S_{N_2}}{S_{T_2}} = \frac{S_N}{S_T} = C,
\]

(3)

it follows that, for any value of \( D_R \) and \( D_T \)

\[
D_R = \left( \frac{CD_0}{D_P} \right) D_T.
\]

(4)

Equation 3 imposes the restriction that the size \( S_T \) of any fronto-parallel extent in the transparency configuration is a constant proportion of the size \( S_N \) of the corresponding fronto-parallel extent in the physical object simulated. The only case that will be considered in this paper is the situation in which the viewing position and the point source are at fixed distances (although not necessarily the same distances) from the screen. Therefore, \( D_0/D_P \) is a constant, and Equation 4 states that the distance from O of any point on the simulated configuration is directly proportional to the distance from P of a corresponding point on the transparency configuration. It follows from Equations 3 and 4 that

\[
L_R = \left( \frac{CD_0}{D_P} \right) L_T.
\]

(5)
where \( L_r = D_{r_2} - D_{r_1} \) and \( L_T = D_{T_2} - D_{T_1} \).

Furthermore,

\[
\tan \beta_T = \frac{K_T}{L_T}
\]

(6)

and,

\[
\tan \beta_R = \frac{K_R}{L_R}
\]

(7)

where,

\[
K_T = S_{T_1} - S_{T_2}
\]

(8)

and,

\[
K_R = S_{R_1} - S_{R_2}
\]

(9)

From Equations 3, 5, 6, 7, 8, and 9, it follows that

\[
\tan \beta_R = \left(\frac{D_p}{D_o}\right) \tan \beta_T.
\]

(10)

Equations 3, 5, and 10 express the size and shape relations between the rigid transparency configuration \( e_r, f_r, g_r, h_r \) and the rigid simulated configuration \( e_r, f_r, g_r, h_r \). Values of \( S_r \) and \( S_T \) are related by the constant \( C \), values of \( L_r \) and \( L_T \) by the constant \( C D_0/D_P \) and values of \( \tan \beta_r \) and \( \tan \beta_T \) by the constant \( D_0/D_P \). Equation 4 expresses the relation between the simulated distance \( D_T \) from the observer of a point on the simulated rigid configuration and the distance \( D_r \) from the point source of a corresponding point on the rigid transparency configuration. Values of \( D_r \) along \( OQ \) and \( D_T \) along \( PQ \) are related by the constant \( C D_0/D_P \).

If the apparatus is to simulate rigid objects moving in depth by means of a rigid transparency configuration, it must be demonstrated also that the movement of the rigid transparency configuration toward or away from \( P \) will have a unique relationship to the movement of the rigid simulated object away from \( O \) or toward \( O \). If the successive derivations of \( D_r \) and \( D_T \) with respect to time are considered in Equation 4, it follows that

\[
\frac{d^nD_r}{dt^n} = \left(\frac{C D_0}{D_P}\right) \left(\frac{d^nD_T}{dt^n}\right)
\]

(11)

In Equation 11, \( d \) indicates derivatives not distances, \( t \) indicates time not transparencies, \( d^nD_r/dt^n \) and \( d^nD_T/dt^n \) indicate any derivative of \( D_r \) and \( D_T \) respectively with respect to time \( (t) \), and \( C D_0/D_P \) is a constant. According to Equation 11, any velocity \( (dD_T/dt) \) or acceleration \( (d^2D_T/dt^2) \), etc., in the movement of the rigid transparency configuration will produce a proportional although not necessarily an identical velocity \( (dD_r/dt) \) or acceleration \( (d^2D_r/dt^2) \), etc., in the movement of the simulated rigid object.

An advantage of the moving-object simulator is that it can simulate complex objects moving in distance at any specified rates. Suppose, for example, that the object to be simulated is a playing card presented in a frontopтoparallel position and moving directly toward or away from \( O \). The positive transparency would be produced from photographing a real playing card oriented frontoparallel to the camera. As required by the suit of the playing card, either a black-and-white positive transparency or a transparency containing colors can be used. The positive transparency is mounted \( (e_r, f_r, Figure 1) \) perpendicular to its direction of travel. Some device must be used to prevent light originating from the point source from reaching the rear-projection screen except by passing through the positive transparency. For example, the transparency might be mounted on an aperture cut in an otherwise opaque baffle. The baffle and the attached transparency can be mounted on a cart that is moved back and forth between the point source and the rear-projection screen by means of a mechanical linkage to a motor. The result is that, if the image on the rear-projection screen is viewed in an otherwise dark room from the position \( O \), it will appear as a real playing card having appropriate colors and moving in distance in an otherwise black visual field. The movement of the transparency can be arranged to be under the control of either the experimenter or the observer as required by the experiment. The characteristics of the simulated movement will be related to the characteristics of the movement of the transparency by the constant \( C D_0/D_P \) as shown in Equations 8 and 11.

If the moving transparency is tilted toward or away from the screen to form a constant angle \( \beta_T \) with \( PQ \) its direction of movement, it will simulate a moving playing card at a constant tilt \( \beta_R \). From Equation 10, \( \beta_T \) will equal \( \beta_R \) only when \( D_T = D_0 \). If the photographic transparency is of a flat surface and is produced by pointing the camera along a perpendicular to the surface, the optical expansion pattern involved in approaching the surface at a constant angle can be simulated \(^{2,6}\) by orienting the transparency at the appropriate angle as determined by Equation 10.

The development of the optical expansion pattern of a rigid moving surface at a particular
constant slant is readily achieved by using the appropriate value of $D_p/D_o$; however, the simulation with this apparatus of a rigid surface rotating at a prescribed velocity is not easily achieved unless $D_o = D_p$. This is demonstrated by differentiating $\tan \beta_R$ and $\tan \beta_T$ with respect to time ($t$) in Equation 10. It follows that

$$\frac{d^n \tan \beta_R}{dt^n} = \left(\frac{D_p}{D_o}\right)^n \left(\frac{d^n \tan \beta_T}{dt^n}\right)$$

(12)

where $d^n \tan \beta_R/d t^n$ and $d^n \tan \beta_T/d t^n$ signify any derivative of $\tan \beta_R$ and $\tan \beta_T$, respectively, with respect to time ($t$), and where $D_p/D_o$ is a constant. It follows from Equation 12 that a proportional relation between the transparency and the simulated object occurs with respect to tangents of angles, not with respect to angles. Only when $D_p = D_o$ will angular displacements, velocities, and accelerations be proportional between the transparency and simulated object. For the simulation of linear depth movement of rigid three-dimensional objects or rigid surfaces at a constant slant, the permissible inequality of $D_p$ and $D_o$ and of $S_T$ and $S_R$ has the advantage that the proper selection of these terms can result in the simulation of a large linear movement in depth from a much smaller linear movement of a transparency.

The range of experimental problems to which the moving-object simulator can be applied will be determined by the quality of the image rendition available from the use of point-source projection systems. Several of the factors involved in image sharpness, image brightness, and the permissible complexity of the object simulated will be considered in relation to the type of system illustrated in Figure 1. The sharpness of the image on the screen is increased (a) by covering reflecting surfaces inside the apparatus with black velvet or some other nonreflecting material, (b) by using, whenever possible, glass photographic plates of high contrast for the transparencies, (c) by decreasing the diameter of the point source, and (d) by using an appropriate rear-projection screen. With the proper precautions, a good quality of image can be achieved.

It is clear from Figure 1 that light from the point source will strike different parts of the transparency and the screen at different angles. Since some reflection as well as transmission of light will occur at each surface, and since the amount reflected is a function of the incident angle, the brightness of the image across the screen will not be completely uniform. Also, any point on the transparency not on the line PQ of Figure 1 will occupy different positions on the screen as a function of its distance from the point source. Therefore, the intensity of the image on the screen related to a particular point on the transparency will change as the transparency moves toward or away from the point source. Clearly, this non-uniformity of brightness can be limited by physical conditions that restrict the range of incidence angles on the transparency and screen.

Some linear movements of some three-dimensional objects cannot be simulated by the apparatus schematically illustrated in Figure 1. These are objects with parts or sides that, during the movement of the object, appear or disappear from the view of the observer. In addition, when complex transparencies are used, special care must be taken to insure that the opaque baffle on which the transparency is mounted does not interfere with the production of the image on the screen.
SUMMARY

A set of equations was developed for the simulation on a screen of the movement of an object or surface toward or away from an observer by the movement of a positive photographic transparency of the object or surface away or toward a point source. The general case was developed for simulating objects in which the distance of the observer from the screen was constant but not necessarily equal to the distance of the point source from the screen. Equations were developed relating the dimensions of the rigid transparency to those of the rigid simulated object. These equations, under a wide variety of conditions, permit the simulation of surfaces or objects moving in depth at any designated linear speed or acceleration with respect to the observer.

REFERENCES
